B. Tech Degree V Semester (Supplementary) Examination June 2011

IT/CS/EC/CE/ME/SE/EB/EI/EE/FT 501 ENGINEERING MATHEMATICS IV

(2006 Scheme)

Time : 3 Hours Maximum Marks : 100

PART - A

(Answer *ALL* questions)

(8 x 5 = 40)

I. (a) A random variable X has density function $p(x) = \dfrac{1}{\pi(1+x^2)}, -\infty < x < \infty$. Find the probability that X^2 lies between $\dfrac{1}{3}$ and 1.

(b) Find the probability that in five tosses of a fair die 'the number 3' appears at most once.

(c) Five hundred ball bearings have a mean weight of 142.30 gms and a standard deviation of 8.50 gms. Find the probability that a random sample of 100 bearings selected without replacement will have a mean weight between 140.61 and 141.75 gms.

(d) Define the following terms:

(i) Population parameter and Sample Statistic

(ii) Type 1 and Type II errors

(iii) Level of significance.

(e) Prove that $1+\mu^2\delta^2 = \left(1+\dfrac{\delta^2}{2}\right)^2$.

(f) Using Δ and E, estimate the missing value in the following table.

x:	0	1	2	3	4
y:	1	2	4	–	16

(g) Evaluate $\int_4^{5.2} \log_e x\,dx$ using Simpson's $1/3^{rd}$ rule taking $h = 0.2$

(h) Consider the initial value problem $\dfrac{dy}{dx} = x^2 + y^2; y(0) = 1$. Estimate y when $x = 0.1$ by Taylor series method.

PART – B

(4 x 15 = 60)

II. (a) Derive the mean and variance of Poisson's distribution. (8)

(b) The mean mark on a final examination was 72 and the standard deviation was 9. The top 10% of the students are to receive grade A. What is the minimum mark a student must get in order to receive grade A, if the marks are normally distributed? (7)

OR

III. (a) Fit a curve of the form $y = ae^{bx}$ to the following data by the method of least squares: (8)

X:	0	5	8	12	20
Y:	3	1.5	1	0.55	0.18

(b) If the heights of 300 students are normally distributed with mean 172cm and standard deviation 8cm, how many students have heights (7)

(i) between 164 and 180cm

(ii) equal to 172cm? Assume the measurements to be recorded to the nearest centimeters.

IV. (a) A test of the breaking strengths of six ropes manufactured by a company showed a mean breaking strength of 3515 kg and a standard deviation of 66kg where as the manufacturer claimed a mean breaking strength of 3630 kg. Can we support the manufacturer's claim at a level of 0.05? (7)

(b) The standard deviation of heights of 16 male students chosen at random in a school of 1000 male students is 6.10 cm. Find 95% and 99% confidence limits of the standard deviation for all male students at the school. (8)

OR

V. (a) A sample of 10 measurements of the diameter of a sphere give a mean $X = 111mm$ and a standard deviation $s = 1.5mm$. Find 99% confidence limits for the actual diameter. (7)

(b) The mean life time of a sample of 100 fluorescent light bulbs produced by a company is computed to be 1570 hours with a standard deviation of 120 hours. If μ is the mean life time of all the bulbs produced by the company, test the hypothesis $\mu = 1600$ hours against $\mu \neq 1600$ using a level of significance of 0.05 and 0.01. (8)

VI. (a) Using Lagrange's interpolation formula, find y(10) from the following table. (7)

x	5	6	9	11
y	12	13	14	16

(b) From the following table, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ $at\ x = 1.5$. (8)

x	1.5	2.0	2.5	3.0	3.5	4.0
y	3.375	7.000	13.625	24.000	38.875	59.000

OR

VII. (a) Represent $y = x^4 + 3x^3 - 5x^2 + 6x - 7$ in factorial polynomial and show that $\Delta^5 y = 0$, taking $h = 1$. (7)

(b) The following table gives the values of x and $y = \sqrt{x}$. Using Sterling's formula, find $\sqrt{1.12}$. (8)

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
F(x)	1.0000	1.0242	1.0480	1.0714	1.0944	1.1170	1.1392

VIII.

(a) Solve $\frac{dy}{dx} = xy, y(0) = 1$ to get y for $x = 0.1$ and 0.2 by modified Euler's method. (6)

(b) Solve the elliptic equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ over the square mesh of side 4 units satisfying the boundary conditions. (9)

$u(0, y) = 0$ for $0 \leq y \leq 4$, $u(4, y) = 12 + y$ for $0 \leq y \leq 4$,

$u(x, 0) = 3x$ for $0 \leq x \leq 4$, $u(x, 4) = x^2$ for $0 \leq x \leq 4$

OR

IX. (a) Using Bender-Schmidt method find the solution of $\frac{\partial^2 u}{\partial x^2} = 32\frac{\partial u}{\partial t}$ taking $h = 0.25$ for $t > 0$, $0 < x < 1$ and $u(x, 0) = 0 = u(0, t), u(1, t) = t$. (7)

(b) Apply Runge-Kutta method to find an approximate value of y for $x = 0.2$ in steps of 0.1 if $\frac{dy}{dx} = x + y^2$, given that $y = 1$ when $x = 0$. (8)

***